SN – 253

V Semester B.A./B.Sc. Examination, Nov./Dec. 2014
(2013-14 and Onwards) (Semester Scheme) (NS)
MATHEMATICS – VI

Time : 3 Hours Max. Marks : 100

***Instruction** : Answer **all** questions.*

Answer **any fifteen** questions : **(15×2=30)**

1) Solve $2yzdx + zxdy - xy(1+z)dz = 0$.

2) Verify the condition for integrability $2yzdz + zxdy - xy(1+z)dz = 0$.

3) Form the partial differential equation by eliminating the arbitrary constants $ax^2 + by^2 + z^2 = 1$.

4) Solve $p^2 + q^2 = 1$.

5) Solve $z - px - qy = 2\sqrt{pq}$.

6) Solve $[D^2 - 4DD^1 + 4(D^1)^2]z = 0$.

7) Write the generating function of $P_n(x)$ and hence find $P_n(-1)$.

8) Express the polynomial $2x - 3x^2$ in terms of Legendre polynomials.

9) Show that $J_{-n}(x) = J_n(-x)$.

10) With the help of Jacobi's series show that $\frac{x}{2} = J_1 + 3J_3 + 5J_5 + \ldots$

11) Using the recurrence relation $nP_n(x) = xP_n^1(x) - P_{n-1}^1(x)$. Prove that $\int_0^1 P_n(x)dx = \frac{1}{n+1}P_{n-1}(0)$.

12) Evaluate $\Delta(xe^x)$.

13) Prove that $E = e^{hD}$.

14) Express the polynomial $x^3 + x^2 + x + 1$ as a factorial polynomial (taking $h = 1$).

15) Write the Newton's backward interpolation formula.

16) Evaluate $\int_0^6 \frac{dx}{1+x^2}$ using trapezoidal rule.

17) Explain : (i) Deterministic (ii) Stochastic mathematical models.

18) A population grows at the rate of 5% per year. Formulate a differential equation for this and solve it.

19) In the case of modelling of projectile motion without air resistance find the maximum range on the horizontal.

20) What are the assumptions to be made in getting a partial differential equation model for a vibrating string ?

(4×5=

II. Answer **any four** questions :

1) Verify the condition for integrability and solve $3x^2dx + 3y^2dy - (x^3 + y^3 + e^{2z})dz =$

2) Solve $\frac{dx}{x^2 - yz} = \frac{dy}{y^2 - zx} = \frac{dz}{z^2 - xy}$.

3) Form a partial differential equation given that

$\phi(x^2 + y^2 + z^2, z^2 - 2xy) = 0$.

4) Solve $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial x \partial y} = \sin x \cos 2y$.

5) Solve by Charpit's method.

$pxy + pq + qy = yz$.

6) A string is stretched and fastened to two points l units apart. Motion is started by displacing the string in the form $y = a\sin\frac{\pi x}{l}$ from which it is released at a time $t = 0$. Show that the displacement of any point at a distance x from one end at a time t is given by $y(x,t) = a\sin\left(\frac{\pi x}{l}\right)\cos\left(\frac{\pi ct}{l}\right)$.

OR

Solve $(x^2 + y^2)(p^2 + q^2) = 1$ using $x = r\cos\theta$, $y = r\sin\theta$.

Answer **any three** questions : **(3×5=15)**

1) Prove that $P_n^1(x) = xP_{n-1}^1(x) + nP_{n-1}(x)$.

2) Prove that $\int_{-1}^{1} f(x)\, P_n(x) dx = \frac{(-1)^n}{2^n n!} \int_{-1}^{1} f^{(n)}(x)\, (x^2 - 1)^n dx$.

3) Expand $f(x) = \begin{cases} 0 \text{ in } & -1 < x < 0 \\ x \text{ in } & 0 < x < 1 \end{cases}$ in terms of Legendre polynomials.

4) Prove the following

a) $\cos(x \sin\theta) = J_0(x) + 2\sum_{1}^{\infty} J_{2n}(x) \cos 2n\theta$

b) $\sin(x \sin\theta) = 2\sum_{1}^{\infty} J_{2n-1}(x) \sin(2n-1)\theta$.

5) Prove that $J_{-5/2}(x) = \sqrt{\frac{2}{\pi x}} \left[\frac{3}{2} \sin x + \frac{(3 - x^2)}{x^2} \cos x\right]$.

Answer **any four** questions : **(4×5=20)**

1) Given $y_3 = 2$, $y_4 = -6$, $y_5 = 8$, $y_6 = 9$ and $y_7 = 17$, calculate $\Delta^4 y_3$.

2) By separation of symbols prove that

$u_x = u_{x-1} + \Delta u_{x-2} + \ldots + \Delta^{n-1} u_{x-n} + \Delta^n u_{x-n}$.

3) Estimate f(7.5) from the table

x	1	2	3	4	5	6	7	8
f(x)	1	8	27	64	125	216	343	512

4) Using Lagrange's formula for interpolation find f(6) from the following data

x	3	7	9	10
f(x)	168	120	72	63

5) Find $f'(x)$ given

x	0	1	2	3	4
f(x)	1	1	15	40	85

and hence find $f'(0.5)$.

6) Using Simpson's $3/8$ rule obtain an approximate value of $\int_0^{0.3}(2x-x^2)^{1/2}dx$ (taking n = 6).

V. Answer **any three** questions : (3x5=

1) In a culture the bacteria count is 1,00,000. The number is increased by 10% in 2 hours. In how many hours will it cross 2,00,000 if the rate of growth of bacteria is proportional to the number present ?

2) A breeder reactor converts relatively stable uranium – 238 into the isotope plutonium – 239. After 15 years it is determined that 0.043% of the initial amount y_0 of the plutonium has disintegrated. Find the half life of this isotope if the rate of disintegration is proportional to the amount remaining.

3) Form the differential equation of the free damped motion in the case of Mass-Spring-Dashpot and discuss (i) over damped and (ii) critically damped cases.

4) A projectile when thrown at an angle $\tan^{-1}3/4$ falls 40 m short of the target. When it is fired at an angle of 45° it falls 50 m beyond the target. Find the distance of the target (from the point of projection).

5) Find the current I (t) in an RLC – circuit with R = 100 ohms, L = 0.1 henry, $C = 10^{-3}$ farad which is connected to a source of voltage E(t) = 155 sin (377 t).

V Semester B.A./B.Sc. Examination, Nov./Dec. 2014
(Prior to 2013-14) (OS) (Semester Scheme)
MATHEMATICS – VI

Time : 3 Hours　　　　Max. Marks : 90

Instructions : *1) Answer **all** questions.*
*2) Notations have usual **significance**.*

I. Answer **any fifteen** questions : **(15×2=30)**

1) Form the partial differential equation by eliminating arbitrary constants $z = (x + a)(y + b)$

2) Solve $p^2 + q^2 = 3$.

3) Solve $z = pq$.

4) Solve $p - x^2 = q + y^2$.

5) Solve $\left(D^2 - 4DD^1 + 4(D^1)^2\right)z = 0$.

6) Prove that $E\nabla = \nabla E = \Delta$.

7) Evaluate $\int_0^3 \frac{dx}{(1+x)^2}$ by Simpson's $\frac{1}{3}^{rd}$ rule by dividing the interval into three equal parts.

8) Find x, when y = 0.3 using Lagrange's inverse interpolation from the data.

x	0.4	0.6	0.8
y	0.3683	0.3332	0.2897

9) Find the missing term from the table

x	2	3	4	5	6
y = f(x)	45	49.2	54.1	–	67.4

10) Write the Newton-Gregory forward interpolation formula.

11) Write any two Newton's Laws of motion.

12) A particle executing SHM has the period 2 secs and amplitude 3 ft, find maximum velocity.

13) If the velocity of projection is 48 ft/sec and the horizontal range is 36 ft find the elevation.

14) A point moves along a curve so that its tangential and normal accelerations are equal, find its velocity.

15) If the radial velocity is proportional to transverse velocity, show that the path is an equiangular spiral.

16) Determine the Law of force for a particle describing the central orbit whose pedal equation is $p^2 = ar$.

17) A particle describes a central orbit $r\theta^2 = 1$, find its velocity.

18) Define Apse and Apsidal distance.

19) Four particles of masses 1, 2, 3 and 4 units of a system have the position vectors. $2t^2\hat{i} + 4t\hat{j} + 5t\hat{k}$, $3t\hat{j} - 2t^2\hat{k}$, $t^2\hat{i} + 2t\hat{k}$ and $4t\hat{i} - 2t\hat{j}$ respectively. Find the kinetic energy of the system.

20) Derive the equation of motion of a single particle of mass 'm' located at the mass centre and acted upon by a force $\vec{f}$.

II. Answer **any three** of the following : (3x5=1

1) Find the partial differential equation by eliminating the function f from $z = e^{ax+by} \cdot f(ax - by)$.

2) Solve $x^2p^2 = yq^2 = a$.

3) Solve by Charpit's method $(p^2 + q^2)x = pz$.

OR

Reduce $\dfrac{\partial^2 z}{\partial x^2} + x^2 \dfrac{\partial^2 z}{\partial y^2} = 0$ to a Cannonical form.

4) Solve $\left(D^2 - 2DD^1 + (D^1)^2\right)z = 12xy$.

5) Solve $4\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}=3u$ given $u(0, y) = 2.e^{5y}$ by the method of separation of variables.

6) Find and discuss various possible solutions of the wave equation $\frac{\partial^2 u}{\partial t^2}=C^2\frac{\partial^2 u}{\partial x^2}$.

OR

Solve $\left(D^2-DD^1+2(D^1)^2+2D^1\right)z=e^{2x+3y}$.

III. Answer **any three** of the following : **(3×5=15)**

1) a) Obtain the function whose FIRST differences is $3x^2+9x+4$.

b) Evaluate $\left(\frac{\Delta^2}{E}\right)X^3$.

2) Prove that

$$u_0+u_1x+u_2x^2+\ldots\text{to }\infty=\frac{u_0}{1-x}+x\frac{\Delta u_0}{(1-x^2)}+x^2\frac{\Delta^2 u_0}{(1-x)^3}+\ldots\text{ to }\infty.$$

3) Using Newton's divided difference formula estimate f(6) from the following.

x	5	7	11	13	21
f(x)	150	392	1452	2366	9702

4) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ at x = 1 from the following.

x	1	2	3	4	5
y	0	6	24	60	120

5) Find the value of $\log_2$, correct to 4 decimal places using trapezoidal rule for $\int_0^4 \frac{dx}{x+4}$ taking h = 1.

IV. Answer **any four** of the following : (4×5=20)

1) Find the workdone by the force field $\vec{F} = (x^2 - y^2)\hat{i} + xy\hat{j}$ along the curve $y = x^3$ from (0, 0) to (2, 8).

2) A particle executes SHM such that it has a speed v when the acceleration is α and a speed u when the acceleration is β. Show that the distance between the 2 positions is $\frac{u^2 - v^2}{\alpha + \beta}$.

3) A cricket ball is thrown with a velocity of 30 m/sec, find the greatest range on the horizontal plane and the two directions in which the ball may be thrown so as to give a range of 45 mts ($g = 10\ m/sec^2$).

4) A particle is projected with a velocity 300 ft/sec at an angle of 60° to the horizontal from the foot of a plane of angle 30°. Find the time of flight and range on the plane ($g = 32\ ft/sec^2$).

5) The angular elevation of the enemy's position on a fort h ft high is β. Show that inorder to shell it the initial velocity of the projectile must not be less than $\sqrt{gh(1 + \operatorname{cosec} \beta)}$.

6) A particle slides down the outside of a smooth vertical circle due to its weight, starting from rest at the highest point. Discuss the motion.

V. Answer **any two** of the following : (2×5=10)

1) Derive the law of force for a particle describes the central orbit whose pedal equation is $pr = a^2$.

2) A particle describes the curve $r^2 = a^2 \cos 2\theta$ under a force 'f' to the pole. Show that $f \alpha \frac{1}{r^7}$.

3) A particle describes the path $r = a \tan \theta$ under a force to the origin. Find its acceleration and velocity interms of 'r'.

4) Three particles of masses 3, 4 and 6 move under the influence of a force so that their position vectors at time 't' are $\vec{r}_1 = 3t\hat{i} - 4\hat{j} + t^2\hat{k}$, $\vec{r}_2 = (t+1)\hat{i} + 4t\hat{j} - 5\hat{k}$ and $\vec{r}_3 = t^2\hat{i} - t\hat{j} + (2t-1)\hat{k}$ respectively. Find (a) The total linear momentum of the system and (b) The total angular momentum of the system about the origin.

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